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Full Wave Analysis of Slot Line and Coplanar Waveguide on a Magnetic Substrate

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Abstract

An analysis is presented for slot line and coplanar waveguide (CPW) on magnetized substrate. The moment method procedure is used to solve for nonreciprocal phase shift of these structures. Plots of calculated and measured phase shift for CPW on ferrite are presented.

I. Introduction

Due to recent progress in the growth and deposition of magnetic films on semiconductors [1], there is renewed interest in the characteristics of printed transmission lines on magnetized substrates. Non-reciprocal devices constructed with planar technology are low cost and are compatible with integrated circuits or other applications which favor coaxial feeds.

This paper describes the full wave analysis of slot line and coplanar waveguide (CPW) on magnetized substrate. Dispersion characteristics of slot lines and CPW on dielectric substrates have been thoroughly investigated [2,3]. Slot line and CPW on magnetic substrates have been measured experimentally by Robinson and Allen [4], Wen [5], and others [6,7]. It has been reported by Robinson and Allen [4] and Harrison et al. [7] that slot line has the inherent advantage of elliptically polarized fields both in the substrate and at the boundaries with air. There are very few papers which deal with the analysis of slot line on ferrite substrates. In particular, Kaneki [8] analyzed slot line and CPW using a variational method. He concluded that microstrip line has better non-reciprocal phase shift than slot line. Robinson and Harrison disagree with the conclusion [4], [7].

In addition to the analysis, measurements of differential phase shift for CPW on a ferrite toroid are presented. The ferrite is latched to its remnant magnetization during the measurements.

II. Theory

Figure 1 shows the geometry of the slot line and CPW under consideration. The analysis of these structures is based upon a spectral domain approach similar to Jackson's [9] and others [10,11]. In what follows, we present an outline of the formulation of a Green's function which relates the

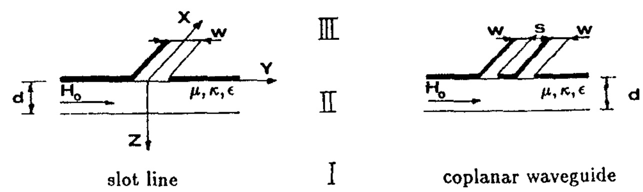

Figure 1. Geometry of slot line and coplanar waveguide.

electric surface currents at $z = 0$ to the electric field in the plane of the slot.

The performance of all ferrite devices is based on their permeability tensor. This tensor, for lossless ferrite transversely magnetized in the plane of the substrate, can be written as

$$\bar{\bar{\mu}} = \mu_o \begin{bmatrix} \mu & 0 & -j\kappa \\ 0 & 1 & 0 \\ j\kappa & 0 & \mu \end{bmatrix} \qquad (1)$$

where $\mu = 1 + \frac{\omega_m \omega_o}{\omega_o^2 - \omega^2}, \kappa = \frac{\omega_m \omega}{\omega_o^2 - \omega^2}$ $\omega_m = \gamma 4\pi M_s, \omega_o = \gamma H_o$, γ is the gyromagnetic ratio, $4\pi M_s$ is the saturation magnetization, and H_o is the impressed D.C. magnetic field. Maxwell's equations for time harmonic fields are written in this case as,

$$\nabla \times \bar{H} = j\omega\epsilon\bar{E} \qquad \nabla \times \bar{E} = j\omega\bar{\bar{\mu}} \cdot \bar{H}$$
$$\nabla \cdot (\bar{\bar{\mu}} \cdot \bar{H}) = 0 \qquad \nabla \cdot \bar{E} = 0 \qquad (2)$$

Splitting $\bar{E}$ and $\bar{H}$ into parts which are longitudinal and transverse to the magnetization (y–direction), the following equations result

$$\frac{\partial^2 \tilde{E}_y}{\partial z^2} - (k_x^2 + k_y^2 - \omega^2 \epsilon \frac{\mu^2 - \kappa^2}{\mu})\tilde{E}_y = -j\omega\mu_o k_y \frac{\kappa}{\mu}\tilde{H}_y$$

$$\frac{\partial^2 \tilde{H}_y}{\partial z^2} - (k_x^2 + \frac{\mu_o}{\mu}k_y^2 - \omega^2\mu_o\epsilon)\tilde{H}_y = j\omega\epsilon k_y \frac{\kappa}{\mu}\tilde{E}_y \qquad (3)$$

where $\sim$ denotes the Fourier transform with respect to the x and y directions. The other field components can be expressed in terms of the y–components as

$$\tilde{E}^{\pm} = \frac{-jk_y \nabla^{\pm} \tilde{E}_y \pm \omega(\mu \pm \kappa)\nabla^{\pm}\tilde{H}_y}{k_y^2 - \omega^2 \epsilon(\mu \pm \kappa)}$$

$$\tilde{H}^{\pm} = \frac{-jk_y \nabla^{\pm} \tilde{H}_y \mp \omega\epsilon\nabla^{\pm}\tilde{E}_y}{k_y^2 - \omega^2 \epsilon(\mu \pm \kappa)} \qquad (4)$$

where

$$\tilde{E}^{\pm} = \tilde{E}_z \pm j\tilde{E}_x, \qquad \tilde{H}^{\pm} = \tilde{H}_z \pm j\tilde{H}_x$$

$$\nabla^{\pm} = \frac{\partial}{\partial z} \mp k_x \qquad (5)$$

If $k_y \neq 0$ (3) can be solved by assuming that

$$\tilde{E}_y = j\eta\tilde{H}_y \qquad (6)$$

and choosing η such that both $\tilde{E}_y$ and $\tilde{H}_y$ have the same z dependence. The resulting equation in η is written as

$$k_y^2 - \omega^2\epsilon\frac{\mu^2-\kappa^2}{\mu} - \omega\mu_o\frac{\kappa}{\mu\eta} = \frac{\mu_o}{\mu}k_y^2 - \omega^2\mu_o\epsilon - \omega\epsilon k_y\frac{\kappa}{\mu}\eta \quad (7)$$

which has two roots, η_n and η_p. $\tilde{E}_y$ and $\tilde{H}_y$ can then be written in the form,

$$\tilde{E}_y = j\eta_p[A_1 \sinh(k_p z) + B_1\cosh(k_p z)] + j\eta_n[A_2\sinh(k_n z) + B_2\cosh(k_n z)]$$

$$\tilde{H}_y = A_1\sinh(k_p z) + B_1\cosh(k_p z) + A_2\sinh(k_n z) + B_2\cosh(k_n z) \qquad (8)$$

where A_1, B_1, A_2, B_2 are arbitrary constants and

$$k_p^2 = k_x^2 + \frac{\mu_o}{\mu}k_y^2 - \omega^2\mu_o\epsilon - \eta_p\omega\epsilon\, k_y\frac{\kappa}{\mu}$$

$$k_n^2 = k_x^2 + \frac{\mu_o}{\mu}k_y^2 - \omega^2\mu_o\epsilon - \eta_n\omega\epsilon\, k_y\frac{\kappa}{\mu}. \qquad (9)$$

This solution is similar to that given in [12]. The fields in the air (region I and III) are simply expressed as

$$\tilde{E}_y = C_1 e^{-k_2 z}, \qquad \tilde{H}_y = C_2 e^{-k_2 z} \qquad (10)$$

and

$$k_2^2 = k_x^2 + k_y^2 - \omega^2\mu_o\epsilon_o \qquad (11)$$

Equations (8), (9) show that there are two eigenwaves describing the fields in a homogeneous magnetized material.

A two dimensional Green's function relating electric fields and electric currents at the $z = 0$ interface is calculated by evaluating the constants in equations (8) and (10) using the appropriate boundary conditions at $z = d$. The numerical solution for these constants in combination with Eqs. (4), (5), (8) allow the expression of $\tilde{H}_x$ and $\tilde{H}_y$ interms of $\tilde{E}_x$ and $\tilde{E}_y$. Finally, the Green's function is formulated by applying the boundary conditions which relate $\tilde{H}_x$, $\tilde{H}_y$ to $\tilde{J}_x$, $\tilde{J}_y$ at $z = 0$.

III. Full Wave Analysis for Propagation Constant

The Green's function is then used in the following expression

$$\begin{bmatrix} J_x(y) \\ J_y(y) \end{bmatrix} = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk_y \begin{bmatrix} \tilde{g}_{xx} & \tilde{g}_{xy} \\ \tilde{g}_{yx} & \tilde{g}_{yy} \end{bmatrix} \begin{bmatrix} \tilde{E}_x(k_y) \\ \tilde{E}_y(k_y) \end{bmatrix} e^{jk_y y} \quad (12)$$

where $\tilde{g}_{ij}$ are functions of β_i, k_y and are the Fourier transforms of the Green's function. An $e^{-j\beta_i z}$ dependence is assumed. The full wave solution is based on the fact that the electric currents must vanish in the slot when β_i is equal to the propagation constant, β. Assuming one pulse expansion mode for E_y in the slot and none for E_x and testing J_y with the same mode yields

$$Z(\beta_i) = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F_e(k_y)|^2 \; \tilde{g}_{yy}(\beta_i, k_y)dk_y \qquad (13)$$

where $F_e(k_y)$ is the Fourier transform of the electric field expansion function. The integral is evaluated numerically and β_i is varied until a zero of Z is found.

Surface wave poles occur in $\tilde{g}_{yy}$. The residues of these poles have been neglected since their effect has been found to be small. The numerical computation of (13) is, however, complicated by the necessity of integrating close to the poles.

IV. Calculated Phase Shift

Fig. 2 shows the calculated value of nonreciprocal phase shift $\Delta\phi = (\beta_f - \beta_r)$ of a slot line, where β_f, β_r are the forward and reverse propagation constants in degrees per centimeter. In comparison with the microstrip line given in [13] the slot line seems to offer higher nonreciprocal phase shift for the same line dimensions and substrates. This result agrees with the conclusions reached in References [4], [7].

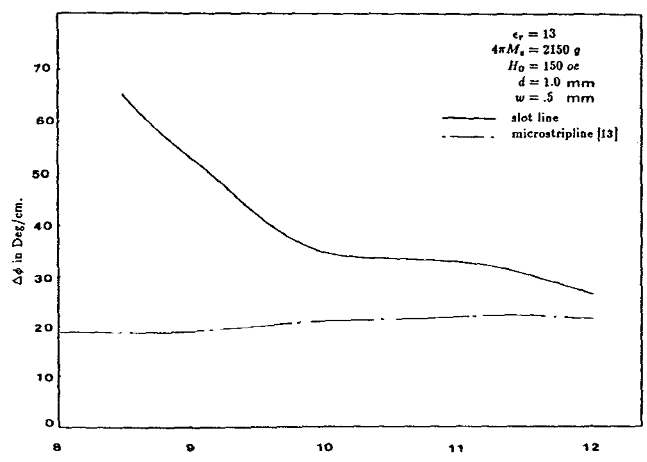

Figure 2. Nonreciprocal phase shift of slot line and microstripline.

For the analysis of CPW, only the odd mode [3] is considered, since this mode can easily be excited by a coaxial feed. A single expansion mode having uniform slot fields has been assumed for simplicity even though it is known to be somewhat inaccurate for closely coupled slots [10].

V. Measured Results

A coplanar waveguide was etched on the metalized surface of a ferrite material (TRANSTECH G-1004) with a toroidal shape (see figure 3). The outer dimensions of the toroid are 9.9 mm × 7.1 mm × 3.17 cm with a wall thickness of 1.59 mm. The dimensions of the CPW are $S = 1.6$ mm and $W = 1.0$ mm which correspond to roughly a 50 ohm impedance. The toroid is latched to the remnant magnetization (493 gauss according to the manufacturers data sheet) which is oriented in the $\pm\hat{y}$ direction. The phase difference between the two states is plotted in figure 4 along with the full wave calculations.

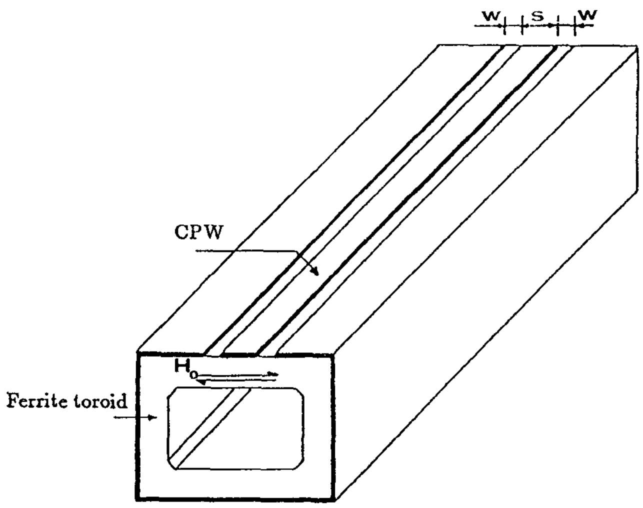

Figure 3. Ferrite toroid with CPW on the surface.

There is reasonable agreement at low frequencies but a large discrepancy occurs at higher frequencies. Possible causes are: (1) The expansion mode used in the full wave expansion was not an accurate enough field representation. (2) The inner walls of the toroid were too close to the CPW structure. (3) The remnant magnetization is not accurately known. Further studies are underway to improve this calculation.

VI. Conclusion

A full wave analysis has been presented for obtaining the dispersion characteristics of slot line and CPW on latched magnetic substrate. One uniform field expansion mode was assumed.

A CPW was etched on the surface of a ferrite toroid and measurements were made of relative phase shifts between two latched states. Reasonable agreement between calculated and measured results occurred at low frequencies but the analysis proved to be inaccurate at higher frequencies. A more accurate analysis is currently under investigation.

Acknowledgement

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References

[1] "Workshop on magnetic materials," IEEE-MTT Symp. 1986.

[2] S. B. Cohn, "Slot line on a dielectric substrate," IEEE Trans . MTT, Vol. MTT-17, pp. 768-778, Oct. 1969.

[3] J. B. Knorr and K. Kuchler, "Analysis of coupled slots and coplanar strips on dielectric substrate," IEEE Trans. MTT, Vol. MTT-23, No. 7, pp. 541-548, July 1975.

[4] G. Robinson and J. Allen, "Slot line application to miniature ferrite devices," IEEE Trans. MTT, Vol. MTT-17, No. 12, pp. 1037-1101, Dec. 1969.

[5] C. Wen, "Coplanar waveguide: a surface strip transmission line suitable for non-reciprocal gyromagnetic device applications," IEEE Trans . MTT, Vol. MTT-17, pp. 1087-1090, Dec. 1969.

[6] L. Courtois and M. Vecchis, "A new class of non-reciprocal components using slot line," IEEE Trans. MTT, Vol. MTT-23, pp. 511-515, June 1975.

[7] G. Harrison, G. Robinson, B. Savage and D. Taft, "Ferrimagnetic parts for microwave integrated circuits," IEEE Trans. MTT, Vol. MTT-19, No. 7, pp. 577-587, July 1971.

[8] T. Kaneki, "Phase constants of the slot line and coplanar waveguide loaded with ferrite substrates," Denshi tsushin gakki ronbunski, pp. 21-22, V55, 1972.

[9] R. W. Jackson and D. M. Pozar, "Full-Wave analysis of microstrip open-end and gap discontinuities," IEEE Trans. MTT, Vol. MTT- 33, No. 10, pp. 1036-1042, Oct. 1985.

[10] R. H. Jansen, "Unified user-oriented computation of shielded , covered and open planar microwave and millimeter-wave transmission-line characteristics," Microwave, Optics and Accoustics, Vol. 3, No. 1, pp. 14-22, Jan. 1979.

[11] T. Itoh, "Spectral domain immitance approach for dispersion characteristics of generalized printed transmission lines," IEEE Trans. MTT, Vol. MTT-28, No. 7, pp. 733-736, July 1980.

[12] F. Lange, "Analysis of shielded strip and slot lines on a ferrite substrate transversely magnetized in the plane of the substrate," Electronics and Communication, AEÜ, Band 36, Hefts, pp. 95-100, March 1982.

[13] E. Riches, P. Brennan, P. Brigginshaw and S. Deeley, "Microstrip ferrite devices using surface field effects for microwave integrated circuits," IEEE Trans., MAG, Vol. MAG-6, pp. 670-673, Sept. 1970.

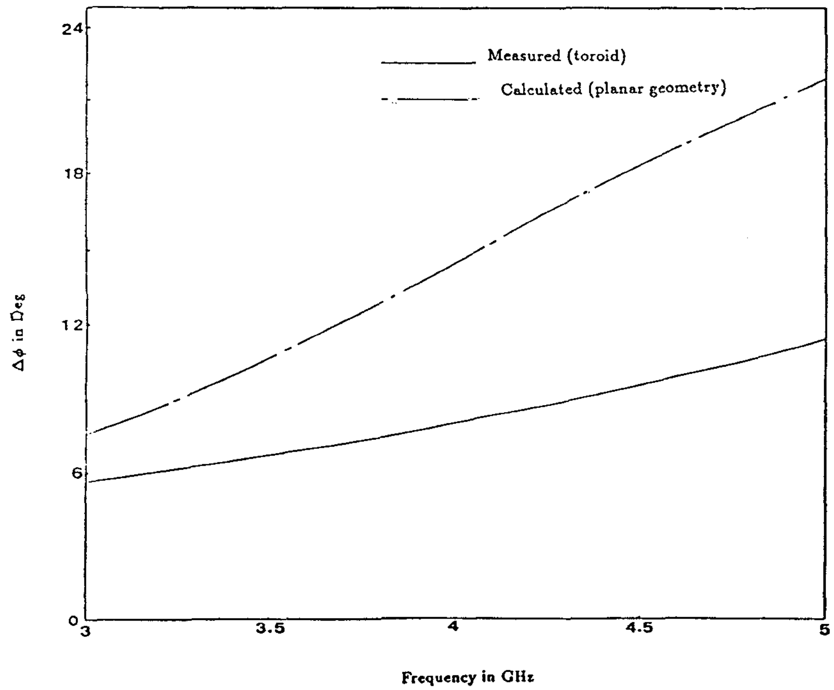

Figure 4. Nonreciprocal phase shift of a coplanar waveguide.